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AN ELECTRIC ARC IN A CHANNEL BEARING A TURBULENT GAS FLOW

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A study of the positive column in an arc in a tube containing a turbulent gas flow is presented. Working formulas are given for estimating the arc parameters.

There are many papers [1-3] on the theory of arc columns stabilized by laminar gas flows; major advances have been made, as is clear from the good agreement between theory and experiment for laminar or near-laminar flow. However, the flow state may differ considerably from laminar in an electric-arc device such as a plasma source, even though the Reynolds number is small, on account of the various types of instability occurring in arcs, which cause fluctuations in the local parameters. According to current views [4], an arc becomes essentially turbulent in the presence of a moderate gas flow from the point where the positive column meets the turbulent boundary layer. The column in such a flow is described by a complicated system of equations difficult to solve. However, various simplifying assumptions can provide solutions that incorporate the major processes, which can enable one to calculate the electrical and thermal characteristics with reasonable accuracy.

1. Formulation and Solution

Here we consider an arc column in a cylindrical channel containing a turbulent gas flow; let $\rho v = \bar{\rho v} + (\rho v')$, $\rho u = (\rho u)'$, $E = \bar{E} + E'$, $S = \bar{S} + S'$, and $\sigma = \bar{\sigma} + \sigma'$, while it is assumed that the assumptions made in [1-3] apply. In the present case, we further specify that the relaxation times of the elementary processes in the plasma are small by comparison with the time scale of the turbulent pulsations. Then the positive column is described by the following equation if we assume that E is constant over the cross section of the tube and neglect turbulent heat transport along the axis, viscous dissipation, the change in the kinetic energy by comparison with the input heat, the convective transfer along the flow, and the radial energy flow arising from heat conduction and turbulence:

$$\frac{\overline{\rho v}\, h_s}{l}\,\frac{\partial \bar{S}}{\partial z} + \overline{\frac{(\rho u)'\, h_s}{R}\,\frac{\partial S'}{\partial r}} = \frac{1}{R^2 r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial \bar{S}}{\partial r}\right) + (\bar{E}^2 + \overline{E'^2})\,\sigma_s \bar{S} - \xi_s \bar{S}, \tag{1}$$

$$\langle I \rangle = 2\pi R^2 \sigma_s \langle E \rangle \int_0^1 \bar{S} r\, dr \tag{2}$$

subject to the conditions

$$\bar{S}(r,\ 0) = \varphi(r),\quad \bar{S}(1,\ z) = 0,\quad \frac{\partial \bar{S}}{\partial r}\,(0,\ z) = 0,\quad \overline{\rho v} = \text{const}. \tag{3}$$

Estimation of the terms in the equations for the arc shows that this model applies for reasonably long channels provided that the discussion is restricted to the region of developed turbulence for low Mach numbers. The instantaneous current and voltage are variable, so Ohm's law in (2) has been written for the effective values. The turbulent pulsations are completely random, so the phase difference between I and E may be taken as zero.

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TABLE 1. The τ Dependence of μ_1 and γ_1

τ	μ_1	γ_1	τ	μ_1	γ_1
0	2,4048	0,2159	—26	0,4592	0,4035
—2	2,1995	0,2327	—28	0,3871	0,4113
—4	2,0000	0,2500	—30	0,3247	0,4182
—6	1,8076	0,2674	—32	0,2711	0,4243
—8	1,6235	0,2846	—34	0,2264	0,4296
—10	1,4487	0,3015	—36	0,1867	0,4343
—12	1,2842	0,3177	—38	0,1543	0,4385
—14	1,1305	0,3332	—40	0,1268	0,4422
—16	0,9888	0,3477	—42	0,1041	0,4455
—18	0,8589	0,3612	—44	0,0852	0,4484
—20	0,7413	0,3735	—46	0,0695	0,4510
—22	0,6357	0,3846	—48	0,0567	0,4533
—24	0,5418	0,3945	—50	0,0461	0,4555

At present, we have no data on the correlation between $(\rho u')$ and dS'/dr in an arc; let the correlation coefficient for the two be K. We assume to a first approximation that the turbulent dissipation is equal to the turbulent transport, namely, $K=\text{const}$, $\sqrt{\overline{(\rho u)'^2}}=\text{const}$; we solve (1) subject to these conditions and show that the standard deviation of the thermal conductivity is approximately

$$\sqrt{\overline{S'^2}} = \frac{\sigma_s R\overline{E'^2}}{K\sqrt{\overline{(\rho u)'^2}}\,h_s}\, r\bar{S}. \tag{4}$$

Then (4) and (1) with the less restrictive condition $K\sqrt{\overline{(\rho u)'^2}}=\text{const}$ gives us that

$$\frac{\partial\bar{S}}{\partial z} = a^2\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\bar{S}}{\partial r}\right) - kr\frac{\partial\bar{S}}{\partial r} + c^2\bar{E}^2\bar{S} - b\bar{S}. \tag{5}$$

We solve (2) and (5) with (3) to get

$$\langle E\rangle = \langle I\rangle\,[F_1^2(4\pi^2R^4\sigma_s^2 + 2c^2\langle I\rangle^2\psi(z))]^{-0.5}, \tag{6}$$

$$\bar{S}(r,\ z) = \frac{\langle I\rangle}{2\pi R^2\sigma_s\langle E\rangle F}\sum_{n=1}^{\infty} A_n\Phi_n(\mu_n,\ r)\exp[-(\mu_n^2a^2+b)\,z], \tag{7}$$

where

$$a^2 = \frac{\pi l}{Gh_s};\quad k = \frac{\pi lR^2\sigma_s\overline{E'^2}}{Gh_s};\quad c^2 = \frac{\pi lR^2\sigma_s}{Gh_s};$$

$$b = \frac{\pi lR^2\xi_s}{Gh_s};\quad \tau = -\frac{2k}{a^2};$$

A_n are the Fourier coefficients in the expansion of $\varphi(r)$ as a series in the function

$$\Phi_n(\mu_n,\ r) = 1+\sum_{m=1}^{\infty}\frac{(-1)^m\mu_n^2(\mu_n^2+\tau)(\mu_n^2+2\tau)\ \dots\ [\mu_n^2+(m-1)\tau]}{2^m(m!)^2}\,r^{2m}$$

over the range $0\le r\le 1$; μ_n are the roots of $\Phi_n(\mu_n,\ 1)=0$; and Φ_n are the eigenfunctions of the following equation, which are bounded at $r=0$:

$$r\frac{d^2\Phi}{dr^2} + \left(1+\frac{\tau r^2}{2}\right)\frac{d\Phi}{dr} + \mu^2r\Phi = 0, \tag{8}$$

where this equation describes the column for $\mu_1^2 = R^2\sigma_s\bar{E}^2 - \xi_sR^2$:

$$F = \sum_{n=1}^{\infty} A_n\gamma_n\exp[-(\mu_n^2a^2+b)\,z];\quad F_1 = F\exp(a^2\tau z);$$

$$\gamma_n = \int_0^1\Phi_n(\mu_n,\ r)\,r\,dr;\quad \psi(z) = \int_0^z F_1^{-2}\,dz.$$

From (6) and (7) we get the mean-mass values of $\bar{S}$ and $\bar{h}$ for the column:

$$\bar{S}_c = \frac{\langle I\rangle}{\pi R^2\sigma_s\langle E\rangle},\quad \bar{h}_c = h_* + \frac{h_s\langle I\rangle}{\pi R^2\sigma_s\langle E\rangle}. \tag{9}$$

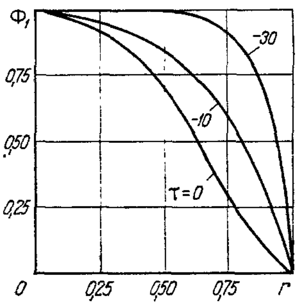

Fig. 1

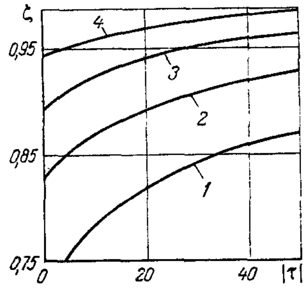

Fig. 2

Fig. 1. Radial distribution of Φ_1.

Fig. 2. Radius of positive column ξ as a function of turbulence parameter τ ($R_1 = 5\cdot10^{-3}$ m); curves 1-4 correspond to $\langle I\rangle = 20$, 40, 80, and 160 A.

The heat loss through the surface as corrected for radiation is

$$q = -2\pi\frac{\partial\bar S}{\partial r}(1,\ z) + \frac{\xi_s\langle I\rangle}{\sigma_s\langle E\rangle}. \tag{10}$$

The quantities μ_n, γ_n, and Φ_n were computed and tabulated; Table 1 gives the values of μ_1 and γ_1 for τ between 0 and -50, which are required in discussing the solution.

2. Particular Cases

2.1. $\varphi(r) = A_1\Phi_1(\mu_1,\ r)$. Simple calculations give

$$\langle E\rangle = \langle E\rangle_\infty\{1-(1-M^2)\exp[-2(\mu_1^2a^2+b-\tau a^2)z]\}^{-0.5}; \tag{11}$$

$$\bar S(r,\ z) = \langle I\rangle\,\Phi_1(\mu_1,\ r)/2\pi R^2\sigma_s\langle E\rangle; \tag{12}$$

$$\bar h_c = h_* + \frac{h_s\langle I\rangle}{\pi R\sigma_s^{0.5}\sqrt{\mu_1^2+R^2\xi_s-\tau}}\{1-(1-M^2)\exp[-2(\mu_1^2a^2+b-\tau a^2)]\}^{0.5}, \tag{13}$$

$$M = \langle E\rangle_\infty/\langle E\rangle_0;\quad \langle E\rangle_\infty = \sqrt{\mu_1^2+R^2\xi_s-\tau}/R\sigma_s^{0.5};$$

$$\langle E\rangle_0 = \langle I\rangle/2\pi R^2\sigma_s A_1\gamma_1.$$

It follows from (11) that $\langle E\rangle$ may increase considerably in the region where the laminar flow gives way to turbulent flow, since here we may have $\langle E\rangle_\infty > \langle E\rangle_0$.

2.2. $\tau = 0$, $\xi_s = 0$. The solution is that derived in the theory of [2]; if $\varphi(r) = A_1J_0(\lambda_1,\ r)$, then

$$E = E_\infty[1-(1-M^2)\exp(-2a^2\lambda_1^2z)]^{-0.5}; \tag{14}$$

$$\bar h_c = h_* + \frac{h_sI}{\pi R\sigma_s^{0.5}\lambda_1}[1-(1-M^2)\exp(-2a^2\lambda_1^2z)]^{0.5}. \tag{15}$$

We see from (11)-(13) and (14-(16) that a turbulent arc tends more rapidly to the limiting situation, since $\mu_1^2 - \tau > \lambda_1^2$.

2.3. In the even more particular case $\tau = 0$, $\varphi(r) = 0$, the solution agrees with the results of [1].

3. Detailed Analysis

A more detailed analysis of the positive column in a turbulent arc may be made in terms of the limiting characteristics for $a^2z\to\infty$; (6)-(8) give

$$\langle E\rangle_\infty = \sqrt{\mu_1^2-\tau+R^2\xi_s}/R\sigma_s^{0.5}; \tag{16}$$

$$\bar S = A_1\Phi_1(\mu_1,\ r);\quad A_1 = \frac{\langle I\rangle}{2\pi R\sigma_s^{0.5}\gamma_1\sqrt{\mu_1^2-\tau+R^2\xi_s}}; \tag{17}$$

$$\bar S_c = S_* + \frac{\langle I\rangle}{\pi R\sigma_s^{0.5}\sqrt{\mu_1^2-\tau+R^2\xi_s}};\quad \left.\frac{d\Phi_1}{dr}\right|_{r=1} = \gamma_1(\tau-\mu_1^2).$$

We see from (16) and (17) that the properties of the column are dependent on the turbulence parameter τ, which is expressed in terms of the standard deviation in the electric field. Figure 2 and Table 1 imply that any increase in $|\tau|$ causes the steady component $\bar{E}=\mu_1/R\sigma_s^{0.5}$ to decrease rapidly, in accordance with the $\mu_1(\tau)$ relation, whereas the effective value $\langle E \rangle$ increases in accordance with (16). Therefore, the distribution of $\bar{S}$ (Fig. 1) and thus of the temperature is substantially modified, with the absolute value of $\bar{S}$ at the boundary of the column increasing, while $\bar{S}_c$ and A_1 fall.

Equation (8) with $\tau=\tau_1=G_1h_s/\pi$ also describes a laminar cylindrical arc in a channel with a distributed flow [5]; then τ_1 is proportional to the gas flow rate G_1, through unit length of the channel, while the gas may be injected ($\tau_1>0$) or withdrawn ($\tau_1<0$). The results of [5] and our solutions show that a turbulent arc is analogous to a laminar arc in a channel from which gas is withdrawn as regards the dependence of $\bar{E}$ and $\Phi_1(\mu_1, r)$ on τ. However, the broadening of the distribution for $\bar{S}$ and the reduction in $\bar{E}$ in a laminar column arise from heat transport from the center to the edge by radial gas flow and also because of the rise in temperature in the column, whereas in our case these effects arise from turbulent heat transport and the increased significance of the fluctuating field component.

Calculations show that for a given $|\tau|=\tau_1$ there is no difference between $\bar{S}_c$ and $\langle E\rangle_\infty$ for a laminar arc with injected gas and a turbulent arc; the increase in $\langle E\rangle$ and the reduction in $\bar{S}_c$ as τ_1 increases in a laminar arc are due to the cooling of the positive column by the relatively cold gas entering through the side of the column and mixing by radial flow. In the present case, the effects occur because of the turbulent gas mixing within the column and the rapid heat transfer to the wall.

On this basis we can interpret $-\tau\pi/h_s$ as the mass G_* of gas mixed into the positive column due to turbulent pulsation and thus responsible for raising the electric field and reducing the arc temperature, the effects being those due to influx of a mass of gas $\tau_1\pi/h_s$ arising from the averaged radial flow. Therefore, there is a relationship between our turbulence parameter τ and the gasdynamic characteristics of the flow, in particular, the flow rate or Reynolds number Re and the degree of turbulence $\varepsilon=\sqrt{\overline{u'^2}}/\bar{v}$.

At high temperatures, such as occur in the positive column, the density fluctuations are fairly small, since $d\rho/dT \sim T^{-2}$; therefore, with sufficient accuracy we can put

$$\varepsilon=\sqrt{\overline{(\rho u)'^2}}/\overline{\rho v}.$$

Then

$$\tau=-2R\varepsilon h_s\int_0^1 \overline{\rho v}\,dr.$$

In the case $\overline{\rho v}=\text{const}$, as is assumed here and in [1–3],

$$\tau=-\mu^* h_s\varepsilon\,\mathrm{Re}=-\varepsilon h_s\frac{2G_*}{\pi R}. \tag{18}$$

Then (18) enables us to put (16) as

$$\langle E\rangle_\infty=\sqrt{\mu_1^2+\mu^* h_s\varepsilon\,\mathrm{Re}+R^2\xi_s}\,/R\sigma_s^{0.5}. \tag{19}$$

The limiting parameters for a laminar arc are independent of the gas flow rate [1-3, 5] and are equal to the values for a flow-free arc. In our case, on the other hand, (19) shows that the field strength and other characteristics are determined by Re, i.e., by the gas flow rate, as well as by the degree of turbulence. This is one of the major distinctive features of a turbulent positive column.

An arc heater has the arc column occupying only part of the channel; the energy equations for conducting and nonconducting parts of the chamber can be used with the condition for heat-flux continuity and the condition at the wall $\bar{S}=-S_*$ to find the distribution of $\bar{S}$ outside the positive column $(1\le r\le 1/\zeta)$:

$$\bar{S}(r)=A_1\gamma_1(\mu_1^2-\tau)\ln r\zeta-S_* \tag{20}$$

as well as the formula for the dimensionless radius $\zeta=R/R_1$:

$$\frac{1}{\zeta}\ln\frac{1}{\zeta}=\frac{2\pi S_* R_1\sigma_s^{0.5}\sqrt{\mu_1^2-\tau+R_1^2\zeta^2\xi_s}}{\langle I\rangle(\mu_1^2-\tau)}. \tag{21}$$

If ζ is known together with the distributions of (17) and (20), we can find an expression for the mass-mean enthalpy in the arc chamber:

$$\bar{h}_{c\infty}=h_*\zeta^2+\frac{h_s\langle I\rangle}{\pi R_1^2\sigma_s\langle E\rangle}\left[1-(\mu_1^2-\tau)\left(\frac{1}{2}\ln\frac{1}{\zeta}-\frac{1}{4\zeta^2}+\frac{1}{4}\right)\right]. \tag{22}$$

These formulas show that the plasma-flow parameters in such a channel are largely dependent on the column radius; Fig. 2 shows ζ calculated from (21) for $\xi_s=0$ and $d=10^{-2}$ m, which indicates that the radius of

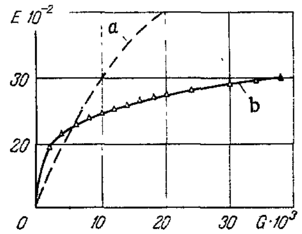

Fig. 3. Field strength $E \cdot 10^{-2}$ V·m^{-1} as a function of gas flow rate G in 10^3 kg·sec^{-1} for $\langle I \rangle = 80$ A and $d = 10^{-2}$ m.

the column tends to unity as τ increases. The electric field also increases, while the mass-mean enthalpy falls and the temperature distribution becomes flattened. Figures 1 and 2 show that the additional turbulence can be utilized to produce a plasma flow with a uniform temperature distribution over much of the cross section.

Curve *a* of Fig. 3 shows the field given by (19) and (21) for $\varepsilon = 10^{-2}$, $\langle I \rangle = 80$ A, and $d = 10^{-2}$ m; we also show (points) the values of E calculated from the following formula from [6]:

$$E = 4.21 \cdot 10^{-2} (10^3 G)(10^{-4} p)^{0.13} (10^2 d)^{-1.02} \varphi,$$
$$\varphi = 355 - I/(10^2 d) + 5.13 \cdot 10^{-3} I^2/(10^2 d)^2, \tag{23}$$

which represents the experimental data for a sectional device for $p = 10^5 - 4 \cdot 10^5$ nm^{-2}, $I = 40-220$ A, $d = 0.5-3 \cdot 10^{-2}$ m, $G = 1.5-24 \cdot 10^{-3}$ kg·sec^{-1}. It is clear that (19) gives a qualitatively correct reflection of the increase in voltage with flow rate even for ε = constant; however, more precise calculation of E requires incorporation of ε(Re), and (19) and (23) allow us to write that

$$\varepsilon = a \, \mathrm{Re}^{-0.7}, \quad a = 2 \cdot 10^{-4} (10^2 d)^{0.26} \varphi^2. \tag{24}$$

A mass $G\xi^2$ flows through the positive column, so we have

$$\tau = -\mu^* h_s \zeta^2 a \, \mathrm{Re}^{0.3}. \tag{25}$$

Curve b of Fig. 3 has been calculated from (19) with (18) and (25). The theoretical curve agrees closely with experiment.

A solution has been obtained for the equations for a turbulent column, and the solution coincides in particular cases with formulas for a laminar arc due to Dautov or Stine and Watson. Some features of turbulence have been established that are not in conflict with experiment; an empirical relationship has been derived for the turbulence of the plasma in an arc heater in relation to the Reynolds number, current, and channel diameter. One expects that these results should be of value in estimating the characteristics of turbulent arcs.

NOTATION

I, current; E, electric field; ρ, σ, $\varkappa$, μ^*, h, S_1, density, electrical conductivity, thermal conductivity, viscosity, enthalpy, and thermal conductivity; R, l, radius and length of positive column; R_1, d, radius and diameter of channel; r, z, cylindrical coordinates along R and l; $S = S_1 - S_*$; S_*, h_*, values of S_1 and h for $r = 1$; $h_S = \partial h/\partial S$, $\sigma_S = \partial\sigma/\partial S$; C, $\bar{C}$, C', $\langle C \rangle$, instantaneous, mean, pulsational, and effective values of C.

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